On the R-sequence and prime key set problem

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This document covers the introduction of the R-sequence, i.e. the sequence of numbers closely related to the distribution of the prime numbers. The paper contains its connection to ζ and Mobius function.

1 Definitions

Definition 1. For natural $n > 1$, we have: $n = \prod_{i=1}^{i=k} {p_i}^{e_i}.\mu$ function is defined as $\Omega(n) = \sum_{i=1}^{i=k} e_i$. From definition we have: $\forall x, y \in N \Omega(xy) = \Omega(x) + \Omega(y)$ and $\forall x, y \in N \Omega(x^y) = y\Omega(x)$.

Definition 2. Ξ matrix is the matrix where each row contains all the numbers $i \in N$-th column contains all consecutive numbers n of $\Omega(n) = i$.

$$\Xi = \begin{bmatrix} 2 & 4 & 8 & 16 & ... & 2^k \\ 3 & 6 & 12 & 24 & ... & 3 * 2^{k-1} \\ 5 & 9 & 18 & 36 & ... & 5\delta(k-1) + 9 * 2^{(k-2)} \\ 7 & 10 & 20 & 40 & ... & 7\delta(k-1) + 10 * 2^{(k-2)} \\ ... & ... & ... & ... & ... & ... \\ \end{bmatrix}$$

Definition 3. From Ξ matrix, one can find R-sequence, ie. sequence r(n)=3,9,10,27,28,30,.. such that $\forall x \in N \exists n x = r(n), m \in N : \forall z \in N \Xi[n+z, m] = 2\Xi[n+z-1, m]$

Definition 4. N-th blocker is the set of numbers within R-sequence from the same column of Ξ matrix. N-blocked are all L-sided to all n-blockers (the column lower than of the requested number; R-sided if the opposite is true, excluding R-sequence) numbers of the same row as all the n-blockers.

Definition 5. Notation $R(n, x, y, z)$ for every $n \in N$ means n with $x, y \in N$ being its coordinates in Ξ and z number of blocker.

2 R-sequence and Mobius function

$\frac{1}{\zeta(s)} = \Pi_p(1 - \frac{1}{p^s}) = \Sigma_{n=1}^{\infty} \frac{\mu(n)}{n^s}$. By definition of Mobius function, we notice that $\forall_{k \in N} \mu(2k+1) = -1$ and $\forall_{k \in N} \mu(2k) = +1$, where k is square-free. Thus, $\zeta(s)^{-1} = \Sigma_{i=1}^{\infty} \Sigma_{j=1}^{\infty} \frac{(-1)^n}{\mu(\Xi[i,j])} - S$, where S defines all non-square-free numbers that appear in sumation. When ζ is important to number theory, we notice how important $R - sequence$ should be. It is straightforward to show that, if we define $\theta(n)$ as the amount of all n-blockers, then $D(n) = \Sigma_{i=1}^{k} \theta(i)$ defines the amount of primes within certain ranges of numbers, defined by R-sequence.

3 Character of R-sequence

The number of n-blockers is likely to be an increasing function, given its connection to ζ shown earlier, but it is not proven. The amount of n-blockers is similar to the one from Fibonacci sequence, but this requires more analysis.

Another interesting aspect concerning Ξ matrix and R-sequence is that when replace i-th column by division of i-th column by $i+1$ column, then the output matrix ξ has its R-sequence containing only $\frac{1}{2}$. All R-sided numbers are $\frac{1}{2}$, but I conjecture that there is exactly no $\frac{1}{2}$ L-sided. This shows clear sign of connection to ζ function.

4 Prime set problem and R-sequence

Let P=2,3,5,7,11,.. be set of primes. Let P_i be subset of set of primes and $p_(n)$ be increasing sequence of prime numbers from set P_i. $p_(n)$ will be referred to as the key. To obtain any information we need to multiply the key vector by the matrix describing any object. The associations of objects and keys with eigenvalues or frequencies in physics seem reasonable.

This model allows perception of the same information in different ways with different keys. It allows incapability of perception of certain signals we don't have keys for. And, thirdly, it is strongly connected with $R-sequence$, which as shown earlier, is strongly connected with the distribution of prime numbers.

Implications of this model to physics (especially to General Theory of Relativity) are omitted in this paper for the sake of simplicity. Assuming every atomic structure can be (together with its geometry) described as vector of prime numbers, this approach seems extremely innovative and promising. For instance, currently we discuss mass in the same context as space-time, whereas mass is defined by the structure of an object (ie. several types of particles). The connection of the former and latter, and the latter and R-sequence will be omitted in this paper.

5 Problems

P0. Find the sequence describing R-sequence.

P1. Are all colosally abundant numbers R-sided?

P2. Take $R(n_1, \mu(n_1), y_1)$, $R(n_2, \mu(n_2), y_2)$ and $m = n_1 n_2$, ie. $R(n_1 n_2, \mu(n_1) + mu(n_2), y_3)$. What is the relation between y_1, y_2, y_3?

P3. Show the distribution of primes in the function of R-sequence.

P4. Let $u(a_1^{k_1} * a_2^{k_2} ... a_n^{k_n}) = k_1 + k_2 + .. + k_n$. Find such sequences f of integer domain of definition such that:$\Omega(f_n) = \Omega(f_{n-1}) + \Omega(gcd(n, f_{n-1}))$. The original equation (without Ω) comes from [3].

P5. 1. Does there exist infinite number of nonsymmetrical polygons with all prime side-lengths inscribed inside a circle? 2. Does there exist infinite number of nonsymmetrical polygons with all prime side-lengths inscribed inside a circle of prime radius?

P6. Knowing that $\forall_{n \in N, n>1} \exists_{M \in N} \forall_{k \in N, k \geq M} w_{k+1}(n) = 2w_k(n)$ by definition of R-sequence, prove that minimal such $w_k(n) = 3^k$ (then $w_{k+1}(n) = w_k(n)$ etc.).

P7. 1. Is there such an operation 'x' that for highly composite and prime gives only prime number? 2. Is there such an operation 'x' that for highly composite and highly composite gives only highly composite numbers?

6 References

[1] Misha Bucko, http://mishabucko.wordpress.com

[2] Grigorij Perelman, Dr., The entropy formula for the Ricci flow and its geometric applications

[3] Eric S. Rowland, Natural prime-generating recurrence, Department of Mathematics, Rutgers University